Digital Audio Signal Processing

DASP

Chapter-3:

Fixed Beamforming

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Overview

- **Introduction & beamforming basics**
- **Data model & definitions**
- **Filter-and-sum beamformer design**
- **Matched filtering**
  - White noise gain maximization
  - Ex: Delay-and-sum beamforming
- **Superdirective beamforming**
  - Directivity maximization
- **Directional microphones** (delay-and-subtract)

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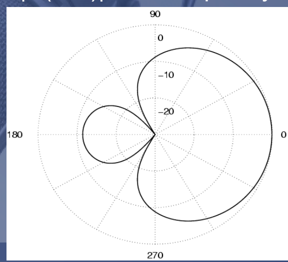
Introduction

- **Directivity pattern of a microphone**
  - A microphone (*) is characterized by a `directivity pattern which specifies the gain & phase shift that the microphone gives to a signal coming from a certain direction (i.e. `angle-of-arrival')
  - In general the directivity pattern is a function of frequency (ω)
  - In a 3D scenario `angle-of-arrival' is azimuth + elevation angle
  - Will consider only 2D scenarios for simplicity, with one angle-of arrival (θ), hence directivity pattern is $H(\omega,\theta)$ (=angle-dependent transfer function)
  - Directivity pattern is fixed and defined by physical microphone design

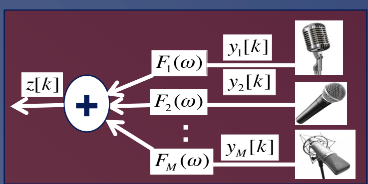
$|H(\omega,\theta)|$ for 1 frequency

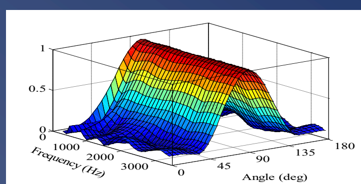


(*) We do digital signal processing, so this includes front-end filtering/A-to-D/..

Introduction

- **Virtual directivity pattern of a microphone array**
  - By weighting or filtering (=freq. dependent weighting) and then summing signals from different microphones, a (software controlled) virtual directivity pattern (=weigthed sum of individual patterns) can be produced





$$F_m(\omega)=F_m(z)\big|_{z=e^{j\omega}}=\sum_{n=0}^{N-1} f_{m,n}.e^{-j\omega n}$$

(=FIR Filters)

$$H_{\text{virtual}}(\omega,\theta)=\sum_{m=1}^{M} F_m(\omega).H_m(\omega,\theta)$$

Introduction

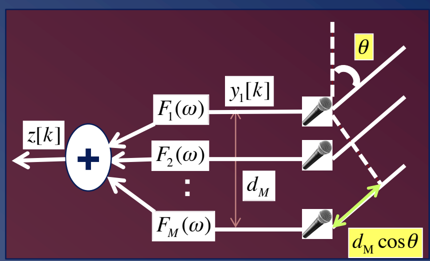
This assumes all microphones receive the same signals (so must be in the same position)

However,
in a microphone array different microphones are in <u>different positions/locations</u>
hence also receive <u>different signals</u>

Introduction

<u>Example</u> : Uniform Linear Array (ULA)

Microphones placed on a line
Uniform inter-micr. distances (d)
Ideal micr. characteristics ($H_m(\omega,\theta)=1$)



For a <u>far-field</u> source signal (i.e. plane wave), each microphone receives the same signal, up to an angle-dependent delay…

$$y_m[k] = y_1[k+\tau_m]$$

$$\tau_m(\theta) = \frac{d_m \cos\theta}{c} f_s \qquad d_m = (m-1)d$$

$$H_{\text{virtual}}(\omega,\theta) = \sum_{m=1}^{M} F_m(\omega).e^{-j\omega\tau_m(\theta)}$$

fs=sampling rate
c=propagation speed=343m/s

Introduction

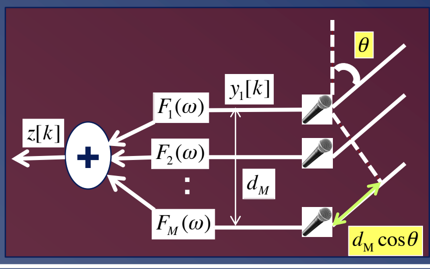
- **Beamforming** = `spatial filtering'

  based on

  microphone characteristics (directivity patterns)

  AND

  microphone array configuration
  (`spatial sampling')



Introduction

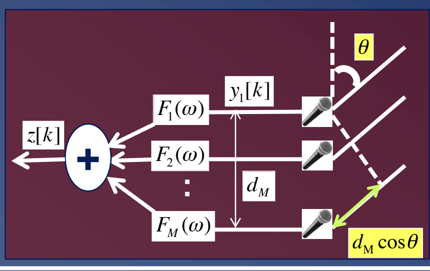
Classification:

Fixed beamforming: data-independent, fixed filters F_m
e.g. delay-and-sum, filter-and-sum

=This chapter

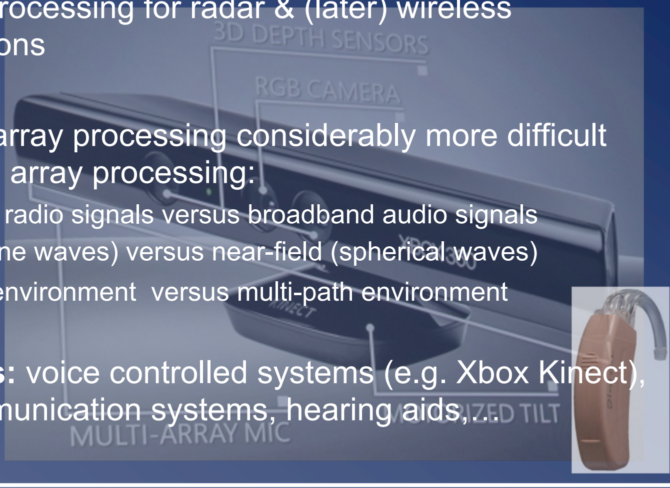
Adaptive beamforming: data-dependent filters F_m
e.g. LCMV-beamformer, generalized sidelobe canceler

=Next chapter



Introduction

- **Background/history**: ideas borrowed from antenna array design and processing for radar & (later) wireless communications



- Microphone array processing considerably more difficult than antenna array processing:
  - narrowband radio signals versus broadband audio signals
  - far-field (plane waves) versus near-field (spherical waves)
  - pure-delay environment versus multi-path environment

- **Applications:** voice controlled systems (e.g. Xbox Kinect), speech communication systems, hearing aids,…

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Data model & definitions 1/5

Data model: source signal in far-field (see p.18 for near-field)

- Microphone signals are filtered versions of source signal $S(\omega)$ at angle θ

$$Y_m(\omega,\theta) = \overbrace{H_m(\omega,\theta)}^{\text{dir. pattern}} . \overbrace{e^{-j\omega\tau_m(\theta)}}^{\text{pos.-dep. phase shift}} . S(\omega)$$

- Stack all microphone signals (m=1..M) in a vector

$$\mathbf{Y}(\omega,\theta) = \mathbf{d}(\omega,\theta).S(\omega)$$

$$\mathbf{d}(\omega,\theta) = \left[H_1(\omega,\theta).e^{-j\omega\tau_1(\theta)} \quad \ldots \quad H_M(\omega,\theta).e^{-j\omega\tau_M(\theta)}\right]^T$$

d is `**steering vector**'

- Output signal after `filter-and-sum' is

H instead of T for convenience (**)

$$Z(\omega,\theta) = \sum_{m=1}^{M} F_m^*(\omega).Y_m(\omega,\theta) = \mathbf{F}^H(\omega).\mathbf{Y}(\omega,\theta) = \{\mathbf{F}^H(\omega).\mathbf{d}(\omega,\theta)\}.S(\omega)$$

Data model & definitions 2/5

Data model: source signal in far-field

$$\mathbf{Y}(\omega,\theta) = \mathbf{d}(\omega,\theta).S(\omega)$$

- If all microphones have the same directivity pattern $H_o(\omega,\theta)$, steering vector can be factored as…

$$\mathbf{d}(\omega,\theta) = \underbrace{H_0(\omega,\theta)}_{\text{dir. pattern}} . \underbrace{\left[1 \quad e^{-j\omega\tau_2(\theta)} \quad \ldots \quad e^{-j\omega\tau_M(\theta)}\right]^T}_{\text{spatial positions}}$$

microphone-1 is used as a reference (=arbitrary)

- Will often consider arrays with **ideal omni-directional microphones** : $H_o(\omega,\theta)=1$

  Example : uniform linear array, see p.6
- Will use microphone-1 as reference (e.g. defining input SNR): $d_1(\omega,\theta)=1$

Data model & definitions 3/5

Definitions:

- In a linear array (p.6) : θ=90º=broadside direction
  θ= 0º =end-fire direction
- Array directivity pattern (compare to p.3)
  = `transfer function' for source at angle θ (-π<θ< π)

$$H(\omega,\theta)=\frac{Z(\omega,\theta)}{S(\omega)}=\mathbf{F}^H(\omega).\mathbf{d}(\omega,\theta)$$

- Steering direction
  = angle θ with maximum amplification (for 1 freq.)

$$\theta_{\max}(\omega)=\arg\max_{\theta}|H(\omega,\theta)|$$

- Beamwidth (BW)
  = region around θ_{max} with amplification > (max.amplif - 3dB) (for 1 freq.)

Data model & definitions 4/5

Data model: source signal + noise

- Microphone signals are corrupted by additive noise

$$\mathbf{Y}(\omega,\theta)=\mathbf{d}(\omega,\theta).S(\omega)+\mathbf{N}(\omega)$$

$$\mathbf{N}(\omega)=[N_1(\omega)\quad N_2(\omega)\quad \ldots \quad N_M(\omega)]^T$$

- Define noise correlation matrix as

$$\mathbf{\Phi}_{noise}(\omega)=E\{\mathbf{N}(\omega).\mathbf{N}(\omega)^H\}$$

- Will assume noise field is homogeneous, i.e. all diagonal elements of noise correlation matrix are equal (=noise power spectrum) :

$$\Phi_{ii}(\omega)=\Phi_{noise}(\omega) \quad , \quad \forall i$$

- Then noise coherence matrix is

$$\mathbf{\Gamma}_{noise}(\omega)=\frac{1}{\phi_{noise}(\omega)}.\mathbf{\Phi}_{noise}(\omega)=\begin{bmatrix}1 & .. & .. \\ .. & \ddots & .. \\ .. & .. & 1\end{bmatrix}$$

Data model & definitions 5/5

Definitions:

- Array Gain = improvement in SNR for source at angle θ ($-\pi < \theta < \pi$)

$$G(\omega,\theta) = \frac{SNR_{output}}{SNR_{input}} = \frac{\left|\mathbf{F}^H(\omega).\mathbf{d}(\omega,\theta)\right|^2}{\mathbf{F}^H(\omega).\mathbf{\Gamma}_{noise}(\omega).\mathbf{F}(\omega)}$$

← |signal transfer function|^2 (with micr-1 used as reference: $d_1 = 1$)
← |noise transfer function|^2

- Will consider 2 special cases...

Data model & definitions 5/5

Definitions:

- Array Gain = improvement in SNR for source at angle θ ($-\pi < \theta < \pi$)

$$G(\omega,\theta) = \frac{SNR_{output}}{SNR_{input}} = \frac{\left|\mathbf{F}^H(\omega).\mathbf{d}(\omega,\theta)\right|^2}{\mathbf{F}^H(\omega).\mathbf{\Gamma}_{noise}(\omega).\mathbf{F}(\omega)}$$

- White Noise Gain =array gain for **spatially uncorrelated** noise

$$WNG(\omega,\theta) = \frac{\left|\mathbf{F}^H(\omega).\mathbf{d}(\omega,\theta)\right|^2}{\mathbf{F}^H(\omega).\mathbf{F}(\omega)}$$

$\Gamma_{noise}^{white} = I$ (e.g. sensor noise)

ps: often used as a measure for robustness

Data model & definitions 5/5

Definitions:

- Array Gain = improvement in SNR for source at angle θ ($-\pi<\theta<\pi$)

$$G(\omega,\theta)=\frac{SNR_{output}}{SNR_{input}}=\frac{\left|\mathbf{F}^H(\omega).\mathbf{d}(\omega,\theta)\right|^2}{\mathbf{F}^H(\omega).\mathbf{\Gamma}_{noise}(\omega).\mathbf{F}(\omega)}$$

- White Noise Gain =array gain for **spatially uncorrelated** noise

$$WNG(\omega,\theta)=\frac{\left|\mathbf{F}^H(\omega).\mathbf{d}(\omega,\theta)\right|^2}{\mathbf{F}^H(\omega).\mathbf{F}(\omega)} \qquad \Gamma_{noise}^{white}=I$$

- Directivity =array gain for **diffuse** noise (=coming from all directions)

$$DI(\omega,\theta)=\frac{\left|\mathbf{F}^H(\omega).\mathbf{d}(\omega,\theta)\right|^2}{\mathbf{F}^H(\omega).\Gamma_{noise}^{diffuse}.\mathbf{F}(\omega)} \qquad \Gamma_{ij}^{diffuse}(\omega)=\text{sinc}\left(\frac{\omega f_s(d_j-d_i)}{c}\right)$$

..without proof

PS: ω is rad/sample (-Π≤ω≤Π)
ω.fs is rad/sec
ω.fs/c = ..=2Π/λ is rad/meter
Zero-crossings at d_j-d_i=(integer).λ/2
for λ=17m (@20Hz)..17mm(@20kHz)

DI(ω,θ_{max}) and WNG(ω,θ_{max}) used as performance criterion

PS: Near-field beamforming

- Far-field assumptions not valid for sources close to microphone array
  - spherical waves instead of plane waves
  - include attenuation of signals
  - 2 coordinates θ,r (=position **q**) instead of 1 coordinate θ (in 2D case)
- Different steering vector (e.g. with $H_m(\omega,\theta)=1$ $m=1..M$) :

$$\mathbf{d}(\omega,\theta) \longrightarrow \mathbf{d}(\omega,\mathbf{q})=\left[a_1 e^{-j\omega\tau_1(\mathbf{q})} \quad a_2 e^{-j\omega\tau_2(\mathbf{q})} \quad \ldots \quad a_M e^{-j\omega\tau_M(\mathbf{q})}\right]^T$$

$$a_m^{\mathbf{e}}=\frac{\left\|\mathbf{q}-\mathbf{p}_{ref}\right\|}{\left\|\mathbf{q}-\mathbf{p}_m\right\|} \qquad \tau_m(\mathbf{q})=\frac{\left\|\mathbf{q}-\mathbf{p}_{ref}\right\|-\left\|\mathbf{q}-\mathbf{p}_m\right\|}{c}f_s$$

e=1 (3D)...2 (2D)

with **q** position of source
$\mathbf{p}_{ref}$ position of reference microphone
$\mathbf{p}_m$ position of m^{th} microphone

PS: Multipath propagation

- In a multipath scenario, acoustic waves are reflected against walls, objects, etc..
- Every reflection may be treated as a separate source (near-field or far-field)
- A more realistic data model is then..

$$\mathbf{Y}(\omega,\mathbf{q}) = \mathbf{d}(\omega,\mathbf{q}).S(\omega) + \mathbf{N}(\omega)$$

$$\mathbf{d}(\omega,\mathbf{q}) = \left[H_1(\omega,\mathbf{q}) \quad H_2(\omega,\mathbf{q}) \quad \ldots \quad H_M(\omega,\mathbf{q})\right]^T$$

with **q** position of source and *Hm*(ω,q), complete transfer function from source position to m-the microphone (incl. micr. characteristic, position, and multipath propagation)

`Beamforming' aspect vanishes here, see also Lecture-4 (`multi-channel noise reduction')

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Filter-and-sum beamformer design

- Basic: procedure based on page 13

$$\underline{H(\omega,\theta)} = \frac{Z(\omega,\theta)}{S(\omega)} = \mathbf{F}^H(\omega).\mathbf{d}(\omega,\theta)$$ ($\mathbf{d}(\omega,\theta)$ assumed known for all ω,θ)

$$\mathbf{F}(\omega) = [F_1(\omega) \;\; \ldots \;\; F_M(\omega)]^T \;\; , F_m(\omega) = \sum_{n=0}^{N-1} f_{m,n}.e^{-jn\omega}$$ (FIR Filters)

Array directivity pattern to be matched to given (desired) pattern $H_d(\omega,\theta)$ over frequency/angle range of interest

- Non-linear optimization for FIR filter design (=ignore phase response)

$$\min_{f_{m,n}, m=1..M, n=0..N-1} \int_\theta \int_\omega \left(|H(\omega,\theta)| - |H_d(\omega,\theta)| \right)^2 \, d\omega \, d\theta$$

- Quadratic optimization for FIR filter design (=co-design phase response)

$$\min_{f_{m,n}, m=1..M, n=0..N-1} \int_\theta \int_\omega \left| H(\omega,\theta) - H_d(\omega,\theta) \right|^2 d\omega \, d\theta$$

Filter-and-sum beamformer design

- Quadratic optimization for FIR filter design (continued)

$$\min_{f_{m,n}, m=1..M, n=0..N-1} \int_\theta \int_\omega \left| H(\omega,\theta) - H_d(\omega,\theta) \right|^2 d\omega \, d\theta$$

$$\mathbf{f}_m = \left[f_{m,0} \;\; \ldots \;\; f_{m,N-1} \right]^T \;\; , \; \mathbf{f} = \left[\mathbf{f}_1^T \;\; \ldots \;\; \mathbf{f}_M^T \right]^T$$ (real-valued)

Kronecker product

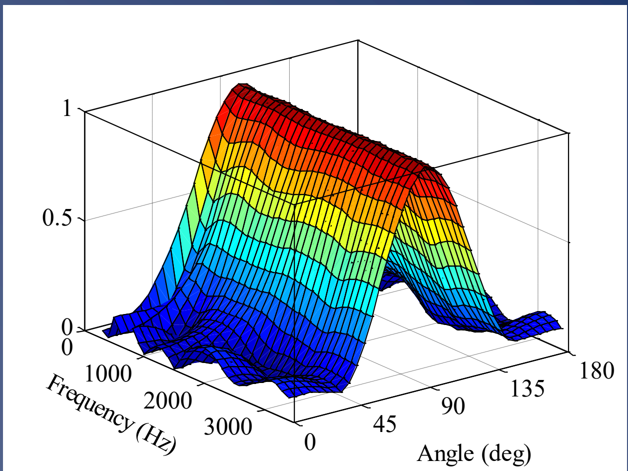
$$H(\omega,\theta) = \mathbf{F}^H(\omega).\mathbf{d}(\omega,\theta) = \left[F_1^H(\omega) \;\; \ldots \;\; F_M^H(\omega) \right].\mathbf{d}(\omega,\theta) = \mathbf{f}^T.\underbrace{(\overbrace{I_{MxM} \otimes \begin{bmatrix} e^{0.j\omega} \\ \vdots \\ e^{(N-1).j\omega} \end{bmatrix}}^{NMxM}).\mathbf{d}(\omega,\theta)}_{\underline{\mathbf{d}}(\omega,\theta)}$$

optimal solution is

$$\mathbf{f}_{optimal} = \mathbf{Q}^{-1}.\mathbf{p} \;\; , \;\; \mathbf{Q} = \int_\theta \int_\omega \underline{\mathbf{d}}(\omega,\theta).\underline{\mathbf{d}}^H(\omega,\theta) \, d\omega \, d\theta \;\; , \;\; \mathbf{p} = \int_\theta \int_\omega \underline{\mathbf{d}}(\omega,\theta).H_d^*(\omega,\theta) \, d\omega \, d\theta$$

Filter-and-sum beamformer design

- Design example



M=8
Logarithmic array
N=50
f_s=8 kHz

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Matched filtering: WNG maximization

- Basic: procedure based on page 17
- Maximize White Noise Gain (WNG) for given angle $\boldsymbol{\psi}$

$$\mathbf{F}^{\text{MF}}(\omega) = \arg\{\max_{\mathbf{F}(\omega)} WNG(\omega,\psi)\} = \arg\{\max_{\mathbf{F}(\omega)} \frac{\left|\mathbf{F}^H(\omega).\mathbf{d}(\omega,\psi)\right|^2}{\mathbf{F}^H(\omega).\mathbf{F}(\omega)}\}$$

- A priori knowledge/assumptions:
  - angle-of-arrival ψ of source signal + corresponding steering vector
  - noise scenario = white

Matched filtering: WNG maximization

- Maximization in $\mathbf{F}^{\text{MF}}(\omega) = \arg\{\max_{\mathbf{F}(\omega)} \frac{\left|\mathbf{F}^H(\omega).\mathbf{d}(\omega,\psi)\right|^2}{\mathbf{F}^H(\omega).\mathbf{F}(\omega)}\}$

  is equivalent to minimization of noise output power (under white input noise), subject to unit response for angle $\boldsymbol{\psi}$, i.e. (**)

$$\min_{\mathbf{F}(\omega)} \mathbf{F}^H(\omega).\mathbf{F}(\omega), \quad \text{s.t.} \quad \mathbf{F}^H(\omega).\mathbf{d}(\omega,\psi) = 1$$

- Optimal solution (`matched filter') is...

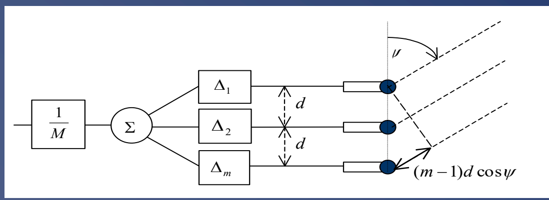
$$\mathbf{F}^{\text{MF}}(\omega) = \frac{1}{\left\|\mathbf{d}(\omega,\psi)\right\|^2}.\mathbf{d}(\omega,\psi)$$

- FIR approximation

$$\min_{f_{m,n}, m=1..M, n=0..N-1} \int_{\omega} \left\|\mathbf{F}^{\text{MF}}(\omega) - \mathbf{F}(\omega)\right\|^2 d\omega$$

Matched filtering example: Delay-and-sum

- Basic: Microphone signals are delayed and then summed together



$$z[k] = \frac{1}{M}\cdot\sum_{m=1}^{M} y_m[k+\Delta_m]$$

$$F_m(\omega) = \frac{e^{-j\omega\Delta_m}}{M}$$

- Fractional delays implemented with truncated interpolation filters (=FIR)
- Consider array with **ideal omni-directional microphones**

Then array can be steered to angle ψ :

$$\mathbf{d}(\omega,\psi) = \begin{bmatrix} 1 & e^{-j\omega\tau_2(\psi)} & \ldots & e^{-j\omega\tau_M(\psi)} \end{bmatrix}^T \qquad \Delta_m = \tau_m(\psi) \quad \Rightarrow \quad \mathbf{F}(\omega) = \frac{\mathbf{d}(\omega,\psi)}{M}$$

Hence (for ideal omni-dir. micr.'s) this is **matched filter solution**

Matched filtering example: Delay-and-sum

ideal omni-dir. micr.'s

- Array directivity pattern $H(\omega,\theta)$:

$$H(\omega,\theta) = \frac{1}{M}\mathbf{d}^H(\omega,\psi).\mathbf{d}(\omega,\theta)$$

$$|H(\omega,\theta)| \leq 1 \quad \text{=destructive interference}$$

$$H(\omega,\theta=\psi) = 1 \quad (\psi = \theta_{\max}) \quad \text{=constructive interference}$$

- White noise gain :

$$WNG(\omega,\theta=\psi) = \frac{\left|\mathbf{F}^H(\omega).\mathbf{d}(\omega,\psi)\right|^2}{\mathbf{F}^H(\omega).\mathbf{F}(\omega)} = .. = M$$

(independent of ω)

For ideal omni-dir. micr. array, delay-and-sum beamformer provides WNG equal to M for all freqs (in the direction of steering direction ψ).

Matched filtering example: Delay-and-sum

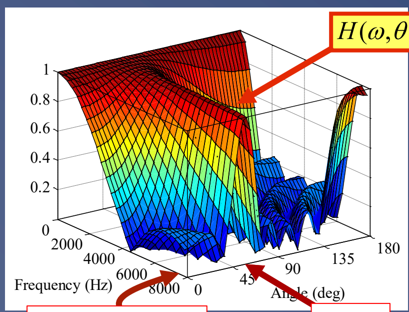
ideal omni-dir. micr.'s

- Array directivity pattern $H(\omega,\theta)$ for **uniform linear array**:

$$H(\omega,\theta) = \sum_{m=1}^{M} e^{-j(m-1)\omega \frac{d(\cos\theta - \cos\psi)}{c} f_s} = \frac{e^{-jM\gamma/2}\sin(M\gamma/2)}{e^{-j\gamma/2}\sin(\gamma/2)}$$

γ

$H(\omega,\theta)$ has sinc-like shape and is frequency-dependent



M=5 microphones
d=3 cm inter-microphone distance
ψ=60° steering direction
f_s=16 kHz sampling frequency

Matched filtering example: Delay-and-sum

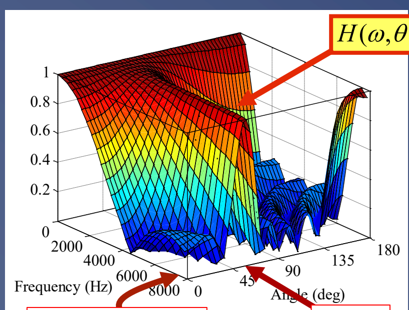
ideal omni-dir. micr.'s

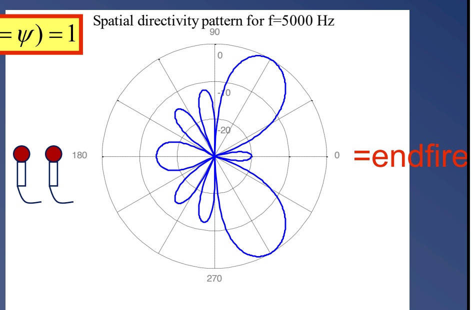
- Array directivity pattern $H(\omega,\theta)$ for **uniform linear array**:

$$H(\omega,\theta) = \sum_{m=1}^{M} e^{-j(m-1)\omega \frac{d(\cos\theta - \cos\psi)}{c} f_s} = \frac{e^{-jM\gamma/2}\sin(M\gamma/2)}{e^{-j\gamma/2}\sin(\gamma/2)}$$

γ

$H(\omega,\theta)$ has sinc-like shape and is frequency-dependent



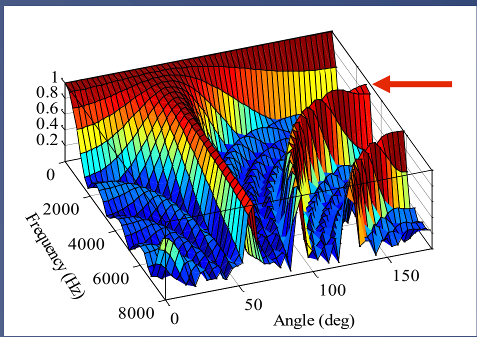


Matched filtering example: Delay-and-sum

ideal omni-dir. micr.'s

For $f \geq \frac{c}{d.(1+|\cos\psi|)}$ an ambiguity, called spatial aliasing, occurs..

This is analogous to time-domain aliasing where now the spatial sampling (=d) is too large.



M=5 microphones
d=8 cm inter-microphone distance
ψ=60° steering direction
f_s=16 kHz sampling frequency

Aliasing does not occur (for any ψ) if $d \leq \frac{c}{f_s} = \frac{c}{2.f_{\max}} = \frac{\lambda_{\min}}{2}$

Matched filtering example: Delay-and-sum

ideal omni-dir. micr.'s

Ignore the

details...

$H(\omega,\theta)=1 \quad iff \quad \gamma = 2\pi.p \quad$ for integer p

1) $\gamma = 0 \quad$ for $\quad \theta = \psi \quad$ (for all ω)

2) if $\psi \leq \frac{\pi}{2}$ then $\gamma = 2\pi$ occurs for $\theta = \pi$ and $f = .. = \frac{c}{d.(1+\cos\psi)}$

3) if $\psi \geq \frac{\pi}{2}$ then $\gamma = 2\pi$ occurs for $\theta = 0$ and $f = .. = \frac{c}{d.(1-\cos\psi)}$

4) etc..

Matched filtering example: Delay-and-sum

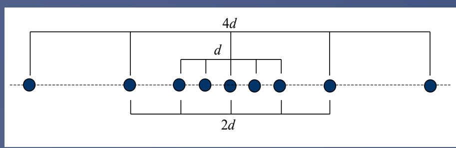
ideal omni-dir. micr.'s

- Beamwidth for a **uniform linear array**:

  $$BW \approx c\frac{\sqrt{96(1-\nu)}}{\omega d M}\sec\psi$$ with e.g. ν=1/sqrt(2) (-3 dB)

  hence large dependence on # microphones, distance (compare p.29 & 31)
  and frequency (e.g. BW infinitely large at DC) (compare to p.23)

- Array topologies:
  - Uniformly spaced arrays
  - Nested (logarithmic) arrays (small *d* for high ω, large *d* for small ω)
  - 2D- (planar) / 3D-arrays



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Super-directive beamforming : DI maximization

- <u>Basic</u>: procedure based on page 17
- Maximize Directivity (DI) for given angle ψ

$$\mathbf{F}^{\mathrm{SD}}(\omega) = \arg\{\max_{\mathbf{F}(\omega)} DI(\omega,\psi)\} = \arg\{\max_{\mathbf{F}(\omega)} \frac{\left|\mathbf{F}^H(\omega).\mathbf{d}(\omega,\psi)\right|^2}{\mathbf{F}^H(\omega).\Gamma_{noise}^{diffuse}.\mathbf{F}(\omega)}\}$$

- A priori knowledge/assumptions:
  - angle-of-arrival ψ of source signal + corresponding steering vector
  - noise scenario = diffuse

Super-directive beamforming : DI maximization

- Maximization in $\mathbf{F}^{\mathrm{SD}}(\omega) = \arg\{\max_{\mathbf{F}(\omega)} \frac{\left|\mathbf{F}^H(\omega).\mathbf{d}(\omega,\psi)\right|^2}{\mathbf{F}^H(\omega).\Gamma_{noise}^{diffuse}.\mathbf{F}(\omega)}\}$

  is equivalent to minimization of noise output power (under diffuse input noise), subject to unit response for angle ψ , i.e. (**)

$$\min_{\mathbf{F}(\omega)} \mathbf{F}^H(\omega).\Gamma_{noise}^{diffuse}(\omega).\mathbf{F}(\omega), \quad \text{s.t.} \quad \mathbf{F}^H(\omega).\mathbf{d}(\omega,\psi) = 1$$

- Optimal solution ('super-directive beamformer') obtained as...

$$\min_{\tilde{\mathbf{F}}(\omega)=[\Gamma_{noise}^{diffuse}(\omega)]^{H/2}.\mathbf{F}(\omega)} \tilde{\mathbf{F}}^H(\omega).\tilde{\mathbf{F}}(\omega), \quad \text{s.t.} \quad \tilde{\mathbf{F}}^H(\omega).[\Gamma_{noise}^{diffuse}(\omega)]^{-1/2}\mathbf{d}(\omega,\psi) = 1$$

$$(\text{with } \Gamma_{noise}^{diffuse}(\omega) = [\Gamma_{noise}^{diffuse}(\omega)]^{1/2}.[\Gamma_{.noise}^{diffuse}(\omega)]^{H/2})$$

$$\Rightarrow \tilde{\mathbf{F}}^{\mathrm{SD}}(\omega) = (\text{factor}).[\Gamma_{noise}^{diffuse}(\omega)]^{-1/2}\mathbf{d}(\omega,\psi) \Rightarrow \mathbf{F}^{\mathrm{SD}}(\omega) = [\Gamma_{noise}^{diffuse}(\omega)]^{-H/2}.\mathbf{F}^{\mathrm{SD}}(\omega) = \ldots$$

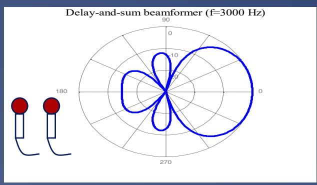
$$\mathbf{F}^{\mathrm{SD}}(\omega) = \frac{1}{\mathbf{d}(\omega,\psi)^H.\{\Gamma_{noise}^{diffuse}(\omega)\}^{-1}.\mathbf{d}(\omega,\psi)}.\{\Gamma_{noise}^{diffuse}(\omega)\}^{-1}.\mathbf{d}(\omega,\psi)$$

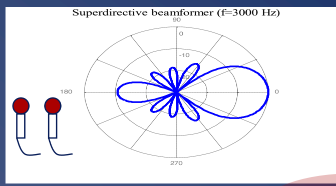
- FIR approximation $\min_{f_{m,n},m=1..M,n=0..N-1} \int_{\omega} \left\|\mathbf{F}^{\mathrm{SD}}(\omega) - \mathbf{F}(\omega)\right\|^2 d\omega$

Super-directive beamforming : DI maximization

ideal omni-dir. micr.'s

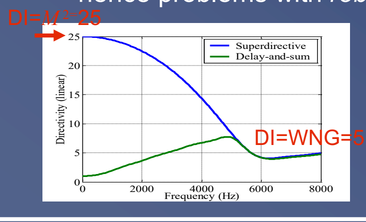
- Directivity patterns for end-fire steering (ψ=0):

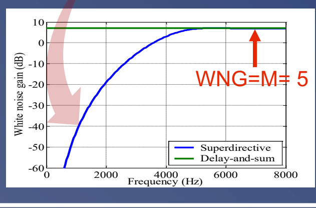




M=5
d=3 cm
f_s=16 kHz

Superdirective beamformer has highest DI, but very poor WNG
(at low frequencies, where diffuse noise coherence matrix becomes ill-conditioned)
hence problems with *robustness* (e.g. sensor noise) !





PS:
diffuse noise ≈ white noise for high frequencies
(cfr. ω→Π and
c/fs=λmin/2≈min(dj-di)
in diffuse noise coherence matrix)

Maximum directivity=M.M obtained for end-fire steering and for frequency->0 (without proof)

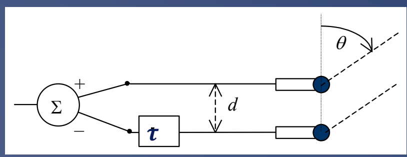
Overview

- **Introduction & beamforming basics**
- **Data model & definitions**
- **Filter-and-sum beamformer design**
- **Matched filtering**
  - White noise gain maximization
  - Ex: Delay-and-sum beamforming
- **Superdirective beamforming**
  - Directivity maximization
- **Directional microphones** (delay-and-subtract)

Differential microphones : Delay-and-subtract

- First-order differential microphone = directional microphone

  2 closely spaced microphones, where one microphone signal is delayed (=hardware) and then subtracted from the other micropone signal



$$H(\omega,\theta)=1-e^{-j\omega(\tau+\frac{d\cos\theta}{c})}$$

Continuous-time system freq.resp. ω = rad/sec

$\omega d/c \ll \pi,\ \omega\tau \ll \pi$

- Array directivity pattern:

$$H(\omega,\theta)\approx j\omega.(\tau+\frac{d}{c}\cos\theta)=\overbrace{j\omega}^{\text{high-pass}}.(\tau+\frac{d}{c}).\overbrace{P(\theta)}^{\text{angle dependence}}$$

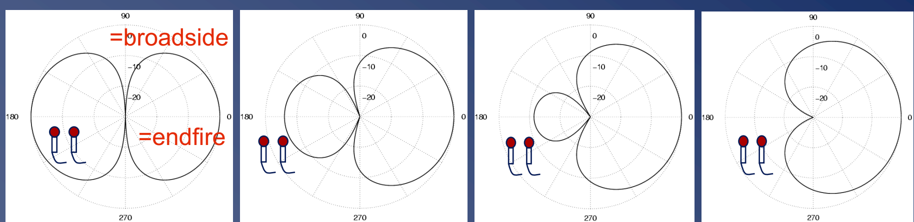
$$\alpha_1=\frac{\tau}{\tau+d/c}$$

$$P(\theta)=\alpha_1+(1-\alpha_1)\cos\theta$$

  – First-order *high-pass* frequency dependence ($j\omega$)
  – $P(\theta)$ = freq.independent (!) directional response
  – $0 \le \alpha_1 \le 1$: $P(\theta)$ is scaled cosine, shifted up with α_1 such that θ_{max} = 0° (=end-fire) and $P(\theta_{max})$=1

Differential microphones : Delay-and-subtract

- Types: dipole, cardioid, hypercardioid, supercardioid (HJ84)



Dipole:
α_1= 0 (τ=0)
zero at 90°
DI=4.8dB

Hypercardioid:
α_1= 0.25
zero at 109°
highest DI=6.0dB

Supercardioid:
α_1= $(\sqrt{3}-1)/2 \approx 0.35$
zero at 125°,
DI=5.7 dB
highest front-to-back ratio

Cardioid:
α_1= 0.5
zero at 180°
DI=4.8dB